Abstract

We provide a framework for registering automatic methods to derive class instances of datatypes, as it is possible using Haskell’s “deriving Ord, Show, . . .” feature.

We further implemented such automatic methods to derive (linear) orders or hash-functions which are required in the Isabelle Collection Framework [1] and the Container Framework [2]. Moreover, for the tactic of Huffman and Krauss to show that a datatype is countable, we implemented a wrapper so that this tactic becomes accessible in our framework.

Our formalization was performed as part of the IsaFoR/CeTA project [3]. With our new tactic we could completely remove tedious proofs for linear orders of two datatypes.
1 Derive manager

theory Derive-Manager
imports Main
keywords print-derives :: diag
  and derive :: thy-decl
begin

⟨ML⟩

The derive manager allows to install various deriving-commands, e.g., to derive orders, pretty-printer, hash-functions, . . . -functions. All of the registered commands are then accessible via the derive-command, e.g., derive hashable list would automatically derive a hash-function for the datatype list.

There is also the diagnostic command print-derives which shows a list of options what can currently be derived.

end

2 Generating linear orders for datatypes

theory Order-Generator
imports Derive-Manager
begin

2.1 Introduction

The order generator registers itself at the derive-manager for the classes ord, order, and linorder. To be more precise, it automatically generates the two functions op ≤ and op < for some datatype dtypr and proves the following instantiations.

• instantiation dtypr :: (ord, . . . , ord) ord
• instantiation dtype :: (order,...,order) order

• instantiation dtype :: (linorder,...,linorder) linorder

All the non-recursive types that are used in the datatype must have similar instantiations. For recursive type-dependencies this is automatically generated.

For example, for the datatype tree = Leaf nat | Node "tree list" we require that nat is already in linorder, whereas for list nothing is required, since for the tree datatype the list is only used recursively.

However, if we define datatype tree = Leaf "nat list" | Node tree tree then list must provide the above instantiations.

Note that when calling the generator for linorder, it will automatically also derive the instantiations for order, which in turn invokes the generator for ord. A later invocation of linorder after order or ord is not possible.

2.2 Implementation Notes

The generator uses the recursors from the datatype package to define a lexicographic order. E.g., for a declaration datatype 'a tree = Empty | Node 'a tree 'a 'a tree this will semantically result in

(Empty < Node _ _ _) = True
(Node 11 12 13 < Node r1 r2 r3) =
  (11 < r1 || 11 = r1 && (12 < r2 || 12 = r2 && 13 < r3))
(_, _) = False
(1 <= r) = (1 < r || 1 = r)

The desired properties (like \[ x < y; y < z \] \[\Rightarrow\] \[ x < z \]) of the orders are all proven using induction (with the induction theorem from the datatype on x), and afterwards there is a case distinction on the remaining variables, i.e., here y and z. If the constructors of x, y, and z are different always some basic tactic is invoked. In the other case (identical constructors) for each property a dedicated tactic was designed.

2.3 Features and Limitations

The order generator has been developed mainly for datatypes without explicit mutual recursion. For mutual recursive datatypes—like datatype a = C b and b = D a a—only for the first mentioned datatype—here a—the instantiations of the order-classes are derived.

Indirect recursion like in datatype tree = Leaf nat | Node "tree list" should work without problems.
2.4 Installing the generator

\textbf{lemma} linear-cases: \((x :: 'a :: linorder) = y \lor x < y \lor y < x\) (proof)

\textbf{ML}

end

3 Hash functions

\begin{verbatim}
theory Hash-Generator
imports ../Collections/Lib/HashCode Derive-Manager
begin

3.1 Introduction

The interface for hash-functions is defined in the class \textit{hashable} which has been developed as part of the Isabelle Collection Framework \cite{1}. It requires a hash-function (\textit{hashcode}), a bounded hash-function (\textit{bounded-hashcode}), and a default hash-table size (\textit{def-hashmap-size}).

The \textit{hashcode} function for each datatype are created by instantiating the recursors of that datatype appropriately. E.g., for \texttt{datatype 'a test = C1 'a 'a | C2 'a test list} we get a hash-function which is equivalent to

\begin{align*}
\text{hashcode (C1 a b)} &= c_1 \times \text{hashcode a} + c_2 \times \text{hashcode b} \\
\text{hashcode (C2 Nil)} &= c_3 \\
\text{hashcode (C2 (a # as))} &= c_4 \times \text{hashcode a} + c_5 \times \text{hashcode as}
\end{align*}

where each \(c_i\) is a non-negative 32-bit number which is dependent on the datatype name, the constructor name, and the occurrence of the argument (i.e., in the example \(c_1\) and \(c_2\) will usually be different numbers.) These parameters are used in linear combination with prime numbers to hopefully get some useful hash-function.

The \textit{bounded-hashcode} functions are constructed in the same way, except that after each arithmetic operation a modulo operation is performed.

Finally, the default hash-table size is just set to 10, following Java's default hash-table constructor.

3.2 Features and Limitations

We get same limitation as for the order generator. For mutual recursive datatypes, only for the first mentioned datatype the instantiations of the \textit{hashable}-class are derived.
3.3 Installing the generator

lemma hash-mod-lemma: \( 1 < (n :: \text{nat}) \implies x \mod n < n \) \( \langle \text{proof} \rangle \)

\( \langle \text{ML} \rangle \)
end

4 Countable datatypes

theory Countable-Generator
imports "~/src/HOL/Library/Countable
Derive-Manager
begin

4.1 Introduction

Brian Huffman and Alexander Krauss have developed a tactic which automatically can prove that a datatype is countable. We just make this tactic available in the derive-manager so that one can conveniently write derive countable some-datatype.

4.2 Features and Limitations

We get similar limitation as for the order generator. For mutual recursive datatypes, only for the first mentioned datatype the instantiations of the countable-class are derived.

4.3 Installing the tactic

There is nothing more to do, then to write some boiler-plate ML-code for class-instantiation.

\( \langle \text{ML} \rangle \)
end

5 Loading derive-commands

theory Derive
imports
  Order-Generator
  Hash-Generator
  Countable-Generator
begin

  We just load the commands to derive (linear) orders, hash-functions, and the command to show that a datatype is countable, so that now all of them
are available. There are further generators available in the AFP entries of lightweight containers and Show.

print-derives

code snippet

6 Examples

theory Derive-Examples
imports
   Derive
   Rat
begin

6.1 Register standard existing types

derive linorder list
derive linorder sum
derive linorder prod

6.2 Without nested recursion

datatype 'a bintree = BEmpty | BNode 'a bintree 'a 'a bintree
derive linorder bintree
derive hashable bintree
derive countable bintree

6.3 Using other datatypes

datatype nat-list-list = NNil | CCons nat list nat-list-list
derive linorder nat-list-list
derive hashable nat-list-list
derive countable nat-list-list

6.4 Explicit mutual recursion

datatype 'a mtree = MEmpty | MNode 'a 'a mtree-list
and 'a mtree-list = MNil | MCons 'a mtree 'a mtree-list
derive linorder mtree
derive hashable mtree
derive countable mtree

6.5 Implicit mutual recursion

datatype 'a tree = Empty | Node 'a 'a tree list
derive linorder tree
derive hashable tree
derive countable tree
6.6 Examples from IsaFoR

datatype (′f,′v)term = Var ′v | Fun ′f (′f,′v)term list
datatype (′f, ′l) lab =
   Lab (′f, ′l) lab ′l
   | FunLab (′f, ′l) lab (′f, ′l) lab list
   | UnLab ′f
   | Sharp (′f, ′l) lab

derive linorder term
derive linorder lab
derive countable term
derive countable lab
derive hashable term
derive hashable lab

datatype (′a, ′b)complex =
   C1 nat ′a ttree
   | C2 (′a, ′b)complex list tree tree ′b (′a,′b)complex (′a,′b)complex2 ttree list
and (′a, ′b)complex2 = D1 (′a,′b)complex ttree

derive linorder complex
derive hashable complex
derive countable complex
end

7 Acknowledgements

We thank

• Lukas Bulwahn and Brian Huffman for the discussion on a generic derive command and the pointer to the tactic for countability.

• Alexander Krauss for pointing me to the recursors of the datatype package.

• Peter Lammich for the inspiration of developing a hash-function generator.
• Andreas Lochbihler for the inspiration of developing generators for the container framework.

• Christian Urban for his cookbook about the ML-level of Isabelle.

• Stefan Berghofer, Cezary Kaliszyk, and Tobias Nipkow for their explanations on several Isabelle related questions.

References

