Lifting Definition Option*

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Abstract

We implemented a command, lift-definition-option, which can be used to easily generate elements of a restricted type \( \{x :: 'a. P x\} \), provided the definition is of the form \( \lambda y_1 \ldots y_n. \text{if check } y_1 \ldots y_n \text{ then Some } (\text{generate } y_1 \ldots y_n :: 'a) \text{ else None } \) and \( \text{check } y_1 \ldots y_n \Rightarrow P \text{ (generate } y_1 \ldots y_n) \) can be proven.

In principle, such a definition is also directly possible using one invocation of lift-definition. However, then this definition will not be suitable for code-generation. To this end, we automated a more complex construction of Joachim Breitner which is amenable for code-generation, and where the test \( \text{check } y_1 \ldots y_n \) will only be performed once. In the automation, one auxiliary type is created, and Isabelle’s lifting- and transfer-package is invoked several times.

This entry is outdated as in the meantime the lifting- and transfer-package has the desired functionality in an even more general way. Therefore, only the examples are kept.

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theory Lifting-Definition-Option-Examples
imports Main
begin

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1 Examples

1.1 A simple restricted type without type-parameters

```c
typedef restricted = \{ i :: int. i mod 2 = 0 \} morphisms base restricted by (intro exI[of - 4]) auto
setup-lifting type-definition-restricted
```

Let us start with just using a sufficient criterion for testing for even numbers, without actually generating them, i.e., where the generator is just the identity function.

```c
lift-definition (code-dt) restricted-of-simple :: int ⇒ restricted option is λ x :: int. if x ∈ \{0, 2, 4, 6\} then Some x else None by auto
```

We can also take several input arguments for the test, and generate a more complex value.

```c
lift-definition (code-dt) restricted-of-many-args :: nat ⇒ int ⇒ bool ⇒ restricted option is λ x y (b :: bool). if int x + y = 5 then Some ((int x + 1) * (y + 1)) else None by clarsimp presburger
```

No problem to use type parameters.

```c
lift-definition (code-dt) restricted-of-poly :: 'b list ⇒ restricted option is λ xs :: 'b list. if length xs = 2 then Some (int (length (xs))) else None by auto
```

1.2 Examples with type-parameters in the restricted type.

```c
typedef ('f restrictedf = \{ xs :: 'f list. length xs < 3 \} morphisms basef restrictedf by (intro exI[of - Nil]) auto
setup-lifting type-definition-restrictedf
```

It does not matter, if we take the same or different type-parameters in the lift-definition.

```c
lift-definition (code-dt) test1 :: 'g ⇒ nat ⇒ 'g restrictedf option is λ (e :: 'g) x. if x < 2 then Some (replicate x e) else None by auto
```

```c
lift-definition (code-dt) test2 :: 'f ⇒ nat ⇒ 'f restrictedf option is λ (e :: 'f) x. if x < 2 then Some (replicate x e) else None by auto
```

Tests with multiple type-parameters.

```c
typedef ('a,'f) restr = \{ (xs :: 'a list, ys :: 'f list) . length xs = length ys \} morphisms base restr
by (rule exI[of - ([], [])], auto)
setup-lifting type-definition-restr
```

```c
lift-definition (code-dt) restr-of-pair :: 'g ⇒ 'e list ⇒ nat ⇒ nat ⇒ ('e,nat) restr option is λ (z :: 'g) (xs :: 'e list) (y :: nat) n. if length xs = n then Some (xs, replicate n y) else None
```
1.3 Example from IsaFoR/CeTA

An argument filter is a mapping $\pi$ from n-ary function symbols into lists of positions, i.e., where each position is between 0 and n-1. In IsaFoR, (Isabelle’s Formalization of Rewriting) and CeTA [1], the corresponding certifier for term rewriting related properties, this is modelled as follows, where a partial argument filter in a map is extended to a full one by means of a default filter.

```plaintext
typedef 'f af = { (\pi :: 'f \times nat \Rightarrow nat list). (\forall f n. set (\pi (f,n)) \subseteq \{0..< n\})}
morphisms af Abs-af by (rule exI[of _], auto)
```

```plaintext
setup-lifting type-definition-af

```plaintext
```plaintext
type-synonym 'f af-impl = (('f \times nat) \times nat list)list

```plaintext
```plaintext
fun fun-of-map-fun :: ('a \Rightarrow 'b option) \Rightarrow ('a \Rightarrow 'b) \Rightarrow ('a \Rightarrow 'b) where
fun-of-map-fun m f a = (case m a of Some b | None => f a)

```plaintext
```plaintext
lift-definition(code-dt) af-of :: 'f af-impl \Rightarrow 'f af option is
\lambda s :: 'f af-impl. if (\forall fidx \in set s. (\forall i \in set (snd fidx). i < snd (fst fidx)))
then Some (fun-of-map-fun (map-of s) (\lambda (f,n). [0..< n])) else None
using map-of-SomeD by (fastforce split: option.splits)
```

1.4 Code generation tests and derived theorems

```plaintext
export-code
restricted-of-many-args
restricted-of-simple
restricted-of-poly
test1
test2
restr-of-pair
af-of
in Haskell

```plaintext
```plaintext
lemma restricted-of-simple-Some:
restricted-of-simple x = Some r \Rightarrow base r = x
using restricted-of-simple.rep-eq[of x]
apply (split if-splits)
apply (simp-all only: option.map option.inject option.simps(3))
done
```

end

3
Acknowledgements

We thank Andreas Lochbihler for pointing us to Joachim’s solution, and we thank Makarius Wenzel for explaining us, how we can go back from states to local theories within Isabelle/ML.

References