Hall’s Marriage Theorem

Dongchen Jiang and Tobias Nipkow

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Abstract

A proof of Hall’s Marriage Theorem due to Halmos and Vaughan [1].

theory Marriage
imports Main
begin

theorem marriage-necessary:
fixes A :: 'a ⇒ 'b set and I :: 'a set
assumes finite I and ∀ i∈I. finite (A i)
and ∃ R. (∀ i∈I. R i ∈ A i) ∧ inj-on R I (is ∃ R. ?R R A & ?inj R A)
shows ∀ J⊆I. card J ≤ card (UNION J A)
⟨proof⟩

The proof by Halmos and Vaughan:

theorem marriage-HV:
fixes A :: 'a ⇒ 'b set and I :: 'a set
assumes finite I and ∀ i∈I. finite (A i)
and ∀ J⊆I. card J ≤ card (UNION J A) (is ?M A I)
shows ∃ R. (∀ i∈I. R i ∈ A i) ∧ inj-on R I
  (is ?SDR A I is ∃ R. ?R R A I & ?inj R A I)
⟨proof⟩

The proof by Rado:

theorem marriage-Rado:
fixes A :: 'a ⇒ 'b set and I :: 'a set
assumes finite I and ∀ i∈I. finite (A i)
and ∀ J⊆I. card J ≤ card (UNION J A) (is ?M A)
shows ∃ R. (∀ i∈I. R i ∈ A i) ∧ inj-on R I
  (is ?SDR A is ∃ R. ?R R A & ?inj R A)
⟨proof⟩

end
References